TECHNICAL NOTE

Prediction of the performance of air-lift pumps

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Equations are developed for predicting the performance of air-lift pumps. The form of the equations allows simple expressions to be obtained for conditions at the 'maximum flow rate condition'

Key words: *pumps, design calculations, efficiency*

Air-lift pumps continue to find a number of industrial applications due to their simplicity, ease of maintenance (as far as the primary circuit is concerned), and their ability to handle corrosive fluids.

This note presents a simple method for use in the design of these devices. The novel aspect of this procedure, compared, for example, to that of Stenning and Martin[1] is in the method of predicting the relative motion of the phases. The method leads to simple equations allowing rapid prediction of mass dryness fraction and total mass flow rate at the 'maximum flow condition'.

This procedure is relevant to turbulent liquid flow; air-lift pump operation under laminar liquid flow is described by Jeelani *et al*[2].

This paper makes use predominantly of the force balance or momentum equation for the two-phase pressure drop in the riser tube. Hussain and Spedding[3] used an entirely different approach based on thermodynamic and bubble energy considerations.

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The design procedure assumes the mixture is incompressible, and with an air density corresponding to the average pressure in the riser.

Two-phase pressure drop

The two-phase pressure drop per unit length in the vertical riser tube is in terms of homogeneous theory for the friction component[4]

$$-\mathrm{D}p = -\mathrm{D}p_{\mathrm{FLO}}\frac{V_H}{V_L} + g\frac{\alpha_L}{V_L} + g\frac{(1-\alpha_L)}{V_G} \tag{1}$$

where the differential operator D indicates derivatives with respect to length, and

$$\frac{V_H}{V_L} = 1 + x\left(\frac{V_G}{V_L} - 1\right) \tag{2}$$

Neglecting gravitational forces on the gas

$$-\mathrm{D}p = -\mathrm{D}p_{\mathrm{FLO}}\frac{V_H}{V_L} + g\frac{\alpha_L}{V_L} \tag{3}$$

The liquid fraction is

$$\alpha_L = \frac{K(1-x)V_L}{K(1-x)V_L + xV_G} \tag{4}$$

Notation

A	Area	Re	Reynolds number
D	Diameter	S	Submergence ratio (Eq (12))
G	Mass velocity	u_G	Gas velocity
g	Gravitational acceleration	u_L	Liquid velocity
K	Velocity ratio, u_G/u_L	V_e	Effective specific volume (Eq (10))
K_e	Effective velocity ratio (Eq (10))	V_G	Specific volume of gas
M	Mass flow rate of mixture	V_H	Homogeneous specific volume (Eq (2))
M_G	Mass flow rate of gas	V_L	Specific volume of liquid
M_L	Mass flow rate of liquid	x	Mass dryness fraction, M_G/M_L
MF	Momentum flux, Mu/A	Z_s	Distance below liquid level in sump of mixer
p	Pressure	Z_0	Distance from mixer to riser outlet
$\mathrm{D}p$	Pressure gradient	α_L	Liquid fraction by volume
$\mathrm{D}p_{\mathrm{FLO}}$	Pressure gradient due to friction if mixture flows as liquid	λ	Friction factor (Eqs (7) and (8))
Δp	Pressure drop to give momentum flux (Eq (9))	μ_L	Absolute viscosity of the liquid

In evaluating the liquid fraction it proves convenient to use Chisholm's[5] equation for the velocity ratio

$$K=\frac{u_G}{u_L}=\left(\frac{V_H}{V_L}\right)^{1/2} \qquad (5)$$

Substituting Eqs (2) and (5) in Eq (4) gives

$$\alpha_L=\frac{1}{\frac{1}{1-x}\left(\frac{V_H}{V_L}\right)^{1/2}+1-\left(\frac{V_L}{V_H}\right)^{1/2}} \qquad (6)$$

The pressure gradient due to friction if the mixture flows as liquid is given by

$$-\mathrm{D}p_{FLO}=\frac{\lambda G^2 V_L}{2D} \qquad (7)$$

and the friction factor can be evaluated from the Blasius equation

$$\lambda=\frac{0.314}{Re^{0.25}}=\frac{0.314\mu_L^{0.25}}{(GD)^{0.25}} \qquad (8)$$

This note is concerned with turbulent flow conditions.

The other major pressure drop in the operation of an air-lift pump is that required to accelerate the two-phase mixture at the mixer. With sufficient accuracy for the present purpose this can be evaluated from

$$\Delta p = MF \qquad (9)$$

which can be evaluated by taking the sum of the momentum fluxes of the liquid and gas phases, which gives

$$MF=G^2V_e=G^2[xV_G+K_e(1-x)V_L]\left[x+\frac{1-x}{K_e}\right] \qquad (10)$$

The effective velocity ratio is evaluated from

$$K_e=K^{0.28} \qquad (11)$$

where K is given by Eq (5). This empirical[6] equation approximates, at the mass dryness fractions of interest in air-lift pumps, to the apparent velocity ratio in the momentum flux measurements of Andeen and Griffith[7] and Wiafe[8].

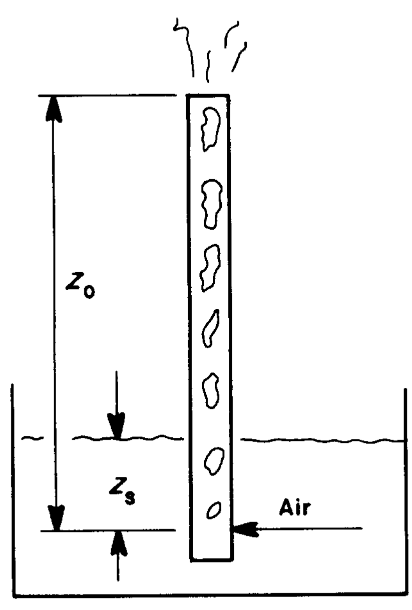

Fig 1 An air lift pump

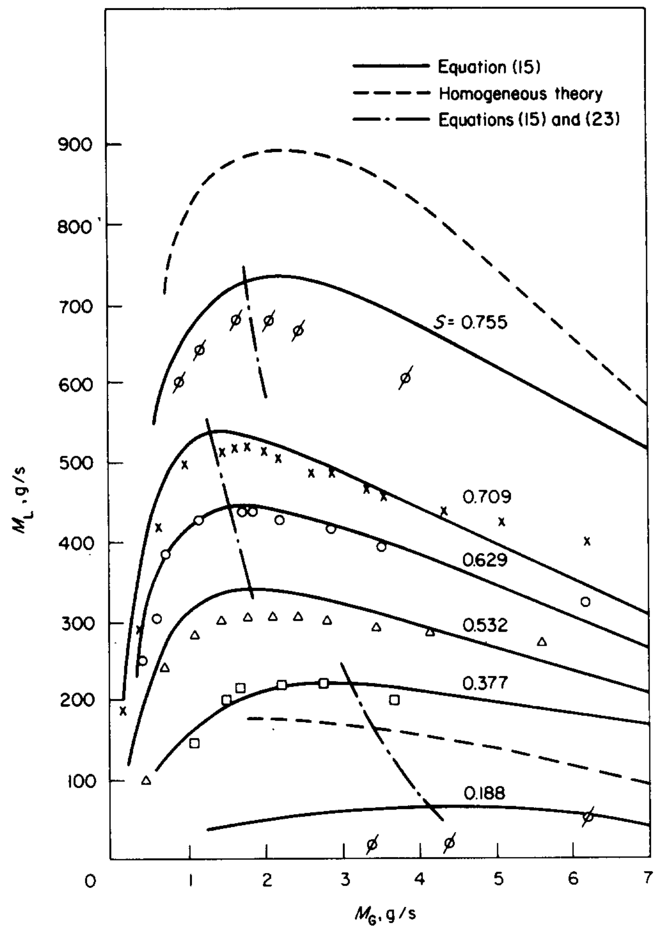

Fig 2 Liquid flow rate to a base of air flow for various submergences. Table 1 gives further details

Submergence ratio

The 'driving head' for the operation of the air-lift pump comes from the submergence Z_s of the air–liquid mixer below the liquid surface. Where Z_0 is the length of the riser tube, the submergence ratio is

$$S=\frac{Z_s}{Z_0} \qquad (12)$$

Fig 1 illustrates the arrangement.

An overall force balance gives

$$gZ_s/V_L=-\mathrm{D}pZ_0+G^2V_e \qquad (13)$$

From Eqs (12) and (13), neglecting the momentum forces,

$$S=-\mathrm{D}pV_L/g \qquad (14)$$

Combining Eqs (3), (12) and (13),

$$-\mathrm{D}p_{FLO}=\frac{g}{V_H}(S-\alpha_L)-\frac{G^2}{Z_0}V_e\frac{V_L}{V_H} \qquad (15)$$

This equation can be solved in conjunction with Eqs (2), (4), (7), (8), (10) and (11) to give the mixture mass flow rate for a given mass dryness fraction. Fig 2

Table 1 Geometric arrangements

Test series	S	D, mm	Z_0, m	References
1	0.755	26.7	8.14	Gosline[9]
2	0.709	25.4	4.0	Stenning and Martin[1]
3	0.629	25.4	4.0	Stenning and Martin[1]
4	0.532	25.4	4.0	Stenning and Martin[1]
5	0.377	26.7	8.14	Gosline[9]
6	0.188	26.7	8.14	Gosline[9]

compares predicted flow rates with experiment. The air-lift pump dimensions are summarised in Table 1. Air–water mixtures at atmospheric pressure were used. The air density was evaluated at the mean pressure in the riser tube. An air density of $1.19\,kg/m^3$ was assumed at a pressure of $10^5\,N/m^2$, and a water density of $10^3\,kg/m^3$. Using Eq (14), this leads to the equation for the gas specific volume

$$\frac{1}{V_G}=1.19\left[1+\frac{9.8Z_0S}{200}\right] \qquad (16)$$

With the exception of the data at the smallest submergence, the predicted flow rates are within ±15%.

Also shown in Fig 2 for the lowest and highest submergences are predictions based on homogeneous theory ($K=1$); the predictions for liquid flow rate are significantly in excess of experiment.

Maximum fluid flow rate

The maximum fluid flow rate for a given diameter can be obtained by differentiating Eq (15) and equating the derivative to zero. This leads to a rather cumbersome equation. A more convenient procedure is as follows. Noting that approximately[10]

$$\frac{V_H}{V_L}=\frac{1}{\alpha_L^2} \qquad (17)$$

then Eq (3) can be approximated.

$$-Dp=-Dp_{FLO}/\alpha_L^2+g\alpha_L/V_L \qquad (18)$$

or

$$-Dp_{FLO}=-Dp\alpha_L^2-\frac{g}{V_L}\alpha_L^3 \qquad (19)$$

Differentiating with respect to α_L, then equating to zero, gives

$$-2Dp\alpha_L-3\frac{g}{V_L}\alpha_L^2=0 \qquad (20)$$

hence using Eq (14)

$$\alpha_L=-\frac{2}{3}\frac{DpV_L}{g}=\frac{2S}{3} \qquad (21)$$

From Eqs (6) and (21), taking $(1-x)$ as unity,

$$\left(\frac{V_H}{V_L}\right)^{1/2}+1-\frac{3}{2S}-\left(\frac{V_L}{V_H}\right)^{1/2}=0 \qquad (22)$$

This is a quadratic equation, hence

$$\left(\frac{V_H}{V_L}\right)^{1/2}=\frac{E\pm(E^2+4)^{1/2}}{2} \qquad (23)$$

where

$$E=\frac{3}{2S}-1 \qquad (24)$$

In Eq (23) the positive value of the square root term is the appropriate solution, as V_H/V_L is always positive. Having evaluated V_H/V_L from Eq (23), the mass dryness is obtained on rearranging Eq (2)

$$x=\frac{\dfrac{V_H}{V_L}-1}{\dfrac{V_G}{V_L}-1} \qquad (25)$$

The mass dryness fraction can also be evaluated from Eqs (17) and (25), but this introduces further errors due to the approximations in the former equation.

Curves obtained using Eqs (15) and (23) are also shown in Fig 2. Table 2 gives the optimum mass dryness fraction obtained using these equations and compares predicted liquid flow rates with experimental values at the maximum flow condition. The maximum difference is 8%.

From Eqs (14), (19) and (21)

$$-Dp_{FLO}=\frac{4g}{27V_L}S^3 \qquad (26)$$

Combining with Eqs (7) and (8) gives an approximate equation for the maximum flow rate for a given submergence

$$G=\left[\frac{8gS^3D^{1.25}}{27\times 0.314\mu_L^{0.25}V_L^2}\right]^{1/1.75} \qquad (27)$$

This approximate equation tends to underpredict the flow rate by the order of 10%. The approximations made in obtaining this equation, in addition to those made in obtaining Eq (15), relate to the use of Eq (17) (in obtaining Eq 21) and to the neglect of momentum forces.

The maximum flow condition may not be the optimum design point. The same flow rates can be obtained with smaller air flow rates with a larger pipe. There is a compromise to be made between running costs and capital investment.

Table 2 Conditions at maximum flow rate

Test series	S	x Eqs (23)/(25)	M_L, kg/s	
			Calc.	Exp.
1	0.755	0.00246	0.740	0.690
2	0.709	0.00258	0.541	0.515
3	0.629	0.00354	0.451	0.440
4	0.532	0.00542	0.345	0.310
5	0.377	0.0134	0.233	0.230
6	0.188	0.0636	0.0665	0.065

Conclusions

Equations for use in the design of air-lift pumps have been developed. Eq (15) for predicting mass flow rates, given the mass dryness fraction, has been shown to give values within 15% of experiment. Approximate equations have also been developed for use at the 'maximum flow rate condition'. Eqs (23) and (25) allow rapid prediction of the mass dryness fraction at that condition, while Eq (27) allows approximate estimates of the associated flow rate.

Appendix

Numerical example

For an air-lift pump of 26.7 mm bore and 8.14 m in length, make an approximate estimate of the maximum liquid flow rate, where the mixer is 3.07 m below the liquid sump level. $V_L = 1\times10^{-3}$ m^3/kg; $\mu_L = 1.002\times10^{-3}$N s/m^2.

Using Eq (12)

$$S = \frac{3.07}{8.14} = 0.3771$$

then from Eq (27) the maximum mixture mass velocity is obtained,

$$G = \left[\frac{8\times9.8\times0.3771^3\times0.0267^{1.25}}{27\times0.314\times(1.002\times10^{-3})^{0.25}\times10^{-6}}\right]^{1/1.75}$$
$$= 362 \text{ kg m}^{-2}\text{ s}^{-1}$$

The corresponding mass dryness fraction is evaluated as follows. From Eq (24)

$$E = \frac{3}{2\times0.3771} - 1 = 2.977$$

then from Eq (23)

$$\left(\frac{V_H}{V_L}\right)^{1/2} = \frac{2.977 + (2.977^2+4)^{1/2}}{2} = 3.282$$

The specific volume of the air is evaluated using Eq (16)

$$\frac{1}{V_G} = 1.19\left[1 + \frac{9.8\times8.14\times0.3771}{200}\right] = 1.368 \text{ kg/m}^3$$

hence the mass dryness fraction is obtained from Eq (25)

$$x = \frac{3.282^2 - 1}{1000/1.368 - 1} = 0.0134$$

The flow cross-section is

$$A = \frac{\pi}{4}\times0.0267^2 = 5.6\times10^{-4}\text{ m}^2$$

The liquid mass flow rate is then

$$M_L = (1-0.0134)\times362\times5.6\times10^{-4} = 200 \text{ g/s}$$

and the corresponding air mass flow rate

$$M_G = 0.0134\times362\times5.6\times10^{-4} = 2.7 \text{ g/s}$$

The mass flow rates in Table 2 were obtained using Eq (15) rather than Eq (26).

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Letter to the Editors

Treatments of thermodynamics

In your March 1982 issue (Vol 3, No 1), Dr T. J. Kotas reviewed my book Thermodynamic Principles of Energy degrading. Many of his criticisms are, I believe, unfounded.

To be quite specific, a horde of text-books have appeared in the last few decades purporting to treat the Second Law in depth, yet almost all of them copy blindly the concepts presented in others, without seeking either to adopt a new approach or simplify the subject for the benefit of practising engineers and scientists. The book is not intended to threaten the narrow world of academics such as Dr Kotas, who I am sure is *au fait* with the latest esoteric treatment of the subject. Frankly, I am not at all interested in the diverse and often confusing treatments of this very important subject which are evident in the literature. My purpose has been to produce a simplified and reliable account of First and Second Law influences, and in doing so I feel I have succeeded in appealing to a large number of practising scientists and research-and-development engineers who earn their keep by productive endeavours in industry.

It is also obvious that the new definitions and simplistic approach have been too threatening for Dr Kotas. Because of the existence of many different definitions, it has been necessary to use synonymous terms. If uncommon meaning has been given to the term 'external irreversibility', I am curious to know